An Experimental Examination of Demand Reduction in Multi-Unit Versions of the Uniform-Price, Vickrey, and English Auctions

I. Introduction

Vickrey (1961) provided a game-theoretic analysis of the single-unit second price auction and showed that bidders have an incentive to set their bids equal to their valuations. Groves (1973), Clarke (1971), and Forsythe, Isaac, and Palfrey (1989) generalized this analysis for public goods and multiple unit auctions. The cornerstone of the Vickrey auction is its pricing rule by which the winner's payment is not based on her own bids but on the bids of other participants. Since a participant's bid has no effect on the price she pays with this pricing rule, she has a dominant strategy to submit bids equal to her true values.[1]

Despite its useful demand-revealing properties, the Vickrey auction is rarely used in practice (see Rothkopf et al. (1990)). A more common auction format used in the field is the Uniform-price auction. It uses the same algorithm as Vickrey to determine winners, but it uses a pricing rule where each winner pays the same price, which is equal to the highest rejected bid. In this auction, bidders with multi-unit demand have an incentive to bid their value only on the first unit and to bid less than their value on subsequent units. Specifically, Noussair (1995) proves, in an auction for two identical goods, that the Symmetric Bayesian Nash Equilibrium has the property that the first bid is revealing and the second is always

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[1] More formally, let j= 1, …, N denote the bidders in the auction. In a multi-unit Vickrey auction, each j submits sealed bids of the form (b^j_k) where k indexes the bid. Given this information the following optimization is performed:

$$\text{Max } \Sigma\, \Sigma\, \delta^j_k\, b^j_k \quad \text{s.t.}$$
$$\Sigma\Sigma\, \delta^j_k \leq \mathbf{M}$$
$$\delta^j_k \in \{0,1\} \quad \forall\ jk$$

where **M** is the number of units to be sold. Let ${}^*\delta^j_k$ denote the solution to the maximization above and let ${}^*b = \Sigma\Sigma\, {}^*\delta^j_k\, b^j_k$. Let –j denote the bids tendered by bidders other than j. To determine the amount j pays in this auction first calculate

$${}^*b^{-j} = \text{Max } \Sigma\, \Sigma\, \delta^{-j}_k\, b^{-j}_k \quad \text{s.t.}$$
$$\Sigma\Sigma\, \delta^{-j}_k \leq \mathbf{M}$$
$$\delta^{-j}_k \in \{0,1\} \quad \forall\ \text{-j and k}$$

Then calculate ${}^*b^j = {}^*b - \Sigma_k\, {}^*\delta^j_k\, b^j_k$, i.e., the optimum without j's winning bids. The amount j pays is then equal to ${}^*b^{-j} - {}^*b^j$. Thus the amount j pays for his winning bids does not depend on what j bids.

under-revealing. The amount of under-revelation depends on the expected profit if a bidder's second bid wins versus the expected profit if the bidder's second bid determines the price. Depending on the values drawn by the participants, Ausubel and Cramton (1998) show that strategic demand reduction can result in inefficient allocations and lower revenue in the Uniform-price auction relative to the Vickrey auction.

Experimentally, Kagel and Levin (1999) test the Uniform-price and Vickrey auctions in which subjects have two-unit non-increasing demands and compete against computerized bidders who demand only one unit and follow the dominant strategy of bidding their value. They observe unambiguous demand reduction on second unit bids in the Uniform-price auction compared to the Vickrey auction.

List and Lucking-Reiley (2000) conducted a novel field experiment in June of 1998 during a sports card show in Orlando, Florida. Stationed at a table at the card show, the authors offered two identical sports cards for sale at auction using either a Uniform-price or Vickrey auction. They conducted 164 auctions (by matching the bids of a randomly selected pair of bidders) with cards ranging in book value from $3 to $70. List and Reiley reach the following conclusions. First, demand reduction occurs more often in the Uniform-price auctions than in the Vickrey auctions. Second, first-unit bids are higher in the Uniform-price auctions relative to the Vickrey auctions, which contradicts theory. Third, when the book price of the sports card is higher, there is a larger percentage reduction in the bids for secondary units in the Uniform-price auctions than in the Vickrey auctions.

The experiment we report complements the List and Reiley field experiment by using a laboratory environment to see if the phenomena they find are consistent when we know the true values of the units to the bidders. We also consider behavior in a variant of the English auction currently used by the Federal Communications Commission (FCC) to allocate spectrum licenses. In this auction buyers submit bids for licenses and then the standing bids for each license are posted. Participants can only submit bids that exceed the standing bid on licenses. The FCC version of this English auction is done in a round-by-round fashion in which bids are submitted simultaneously and then standing bids are announced for the next round. In addition, an activity rule restricts participants to bid on weakly fewer licenses that they bid on in the previous round. The auction stops when no new bids are tendered in a round. This auction form has the following equilibrium properties when identical objects are auctioned: (1) The prices across the objects should be the same and (2) there should be demand reduction by bidders for units greater than their first unit of demand. These two equilibrium properties are the same as those found in the Uniform-price auction, but the English auction and the Uniform-price auction do not necessarily have the same game theoretic outcomes (see Porter and Vragov (2000) for examples). However, in an auction for two units, if there are two demanders, where each has two-unit demands whose values are drawn independently from a uniform distribution then the Bayes-Nash equilibrium in the Uniform-price and English auctions deliver identical outcomes. The equilibrium outcomes are characterized by revelation on the high valued unit and a zero bid on the low valued unit (see Appendices B and C for a proof of this statement).

II. Experimental Design

An equal number of Vickrey, Uniform-price, and English auctions were conducted. Two sessions of each treatment were run with ten participants in each session. Thus a total of 60 subjects were used, 20 for each treatment. Each session consisted of a sequence of 30 auctions. In each auction ten subjects were randomly paired. Each subject was given induced values from three different value ranges. In the first ten auctions, values were drawn between 30 and 170 cents, in the second ten auctions– between 70 and 260 cents, and in the last ten auctions – between 2 and 50 cents. For each value range, 20 value draws were made and put into 10 pairs of values. Values were kept the same for all treatments. Subjects were told that their values would be drawn from these supports for the requisite auctions. The purpose of using three different ranges was motivated by result (3) above reported by List and Reiley. Each subject was then anonymously paired with another subject and participated in an auction for two units. Each subject in a pair received one of the ten value draws without replacement for each auction with the specified value range. For example, with the [30, 170] range, a subject could be faced in the first auction with values of 150 and 45, in the second auction with 160 and 160, in the third auction with 45 and 30, etc. (all subjects had flat or downward sloping demand curves). Every other subject would also receive those values in a different auction. Since the values were "rotated", every subject had a chance to bid on the same ten pairs of values. We did this so we could observe each bidder's bid function on the same set of values (see Appendix A for the actual value assignments for each auction).

We designed the experiments specifically for two people and two units to be allocated in order to "replicate" the List and Reiley design that used two bidders and two sports cards per auction. In this manner, we can use our results in the controlled laboratory setting to illuminate their findings.

In the Vickrey and Uniform-price treatments, each subject could submit at most two bids, one for each of the two units for which they had values. Once submitted the bids were compared with the bids submitted by the subject in their pairing and the results from their auction were shown to them. They could see both their bids and their counterparts' bids, but not the bids of the other paired participants.

The English auction procedure was similar to the process used by the FCC. The price for each auction began at a very low price per unit. The price would rise every 10 seconds by a specific increment if total demand were greater than 2 units (in auctions 1 - 10 and 21 - 30 the initial price and increment was 4; in auctions 11 - 20 the initial price and increment was

10). Subjects were asked to state how many units they wanted (0, 1 or 2) at the given price during a 10-second bidding round. Subjects could never increase their demand in an auction, and they could always see the demand of the other bidder. Thus, if a subject dropped her demand from 2 units to 1 unit that was a clear commitment since she could not increase her demand to 2 units later in that auction. This is akin to the FCC's eligibility rule and helps eliminate punishment strategies that expand the set of Nash equilibria.

We test the following hypotheses:

<u>Hypothesis 1</u>: Bids are demand revealing in the Vickrey auction. Hence, if we let $b_{ij}(v_{ij})$ denote participant i's bid for unit j that has value v_{ij} then:

$$b_{ij}(v_{ij}) = v_{ij} \qquad \forall ij$$

<u>Hypothesis 2</u>: Bids on the high valued units are demand revealing in the Uniform-price auction. That is,

$$b_{i1}(v_{i1}) = v_{i1} \qquad \forall \mathrm{i}$$

<u>Hypothesis 3</u> (strong): If values are drawn from a uniform distribution, then the bids are 0 on the low valued units in the Uniform-price auction. That is,

$$b_{i2}(v_{i2}) = 0 \qquad \forall \mathrm{i}$$

List and Reiley could not directly test the preceding hypotheses because they did not know the actual values or value distributions of their bidders. Instead they use the results of their Vickrey auctions as a proxy for true bidder values. This leads us to examine the following hypothesis:

Hypothesis 4 (weak): Bids for the low valued units in the Uniform-price auction are below the bids on the low valued in the Vickrey auction. That is,

$b^U_{i2}(v_{i2}) < b^V_{i2}(v_{i2}) \qquad \forall i$

where U denotes the Uniform-price auction and V denotes the Vickrey auction.

For the English auction, we can only observe the number of units demanded by each participant at each price increment until the auction closes. Thus, the hypotheses below focus on the final price and allocation in the auction.

Hypothesis 5: If values are drawn from a uniform distribution, then the English auction closes at the opening price and no price increments will be made.

Hypothesis 6: If values are drawn from a uniform distribution, then revenues in the English and Uniform-price auction are equal and lower than revenue in the Vickrey auction. If we let $R(v_1, v_2, v_3, v_4)$ denote revenue when the values drawn are (v_1, v_2, v_3, v_4), then:

$$0 \cong R^U(v_1, v_2, v_3, v_4) = R^E(v_1, v_2, v_3, v_4) < R^V(v_1, v_2, v_3, v_4).$$

Hypothesis 7: If values are drawn from a uniform distribution, then efficiency of the English and of the Uniform-price auctions are equal and lower than efficiency of the Vickrey auction. If we let $E(v_1, v_2, v_3, v_4)$ denote efficiency when the values drawn are (v_1, v_2, v_3, v_4), then:

$$E^U(v_1, v_2, v_3, v_4) = E^E(v_1, v_2, v_3, v_4) < E^V(v_1, v_2, v_3, v_4) = 100\%.$$

Table 1 shows the revenue and efficiency predictions for value draws in our experiments. In the column labeled Value Draw, each two-tuple is the values drawn by the 2 participants in the auction.

Table 1

Nash Equilibrium Revenue (in cents) and Efficiency Predictions across Vickrey, Uniform-price and English Auctions for selected value draws. Since there are only two units for sale, there are only two possible allocations at the end of each auction: 1) One bidder wins both units 2) Each bidder wins a unit. This table lists whether or not the optimal allocation is supported as a Nash Equilibrium outcome.

Value Draw	**Vickrey**		**Uniform-price**		**English**	
	Revenue	Efficient	Revenue	Efficient	Revenue	Efficiency
(160, 160), (48, 33)	81	Yes	0	No	8	No
(150, 45), (160,160)	195	Yes	0	No	8	No
(200, 200), (100, 80)	180	Yes	0	No	20	No
(20, 20), (50, 5)	25	Yes	0	Yes	8	Yes
(160, 160), (140, 30)	170	Yes	0	No	8	No
(200, 200), (160, 140)	300	Yes	0	No	20	No
(50, 4), (20, 20)	24	Yes	0	Yes	8	Yes
(100, 70), (200, 200)	170	Yes	0	No	20	No
Total (All auctions)	40, 370	100%	0	52%	1, 800	52%

List and Reiley's third result suggests that the ratio $\frac{b_2^U(v)}{b_2^V(v)}$ is smaller the larger the book value of the card. Thus we test the following hypothesis:

<u>*Hypothesis 8*</u>*:* $\frac{b_2^U(v)}{b_2^V(v)}$ *will be a decreasing function of the support from which values are drawn.*

III. Experimental Procedure and Summary

The experiments were conducted in the Economic Science Laboratory at the University of Arizona. After reading computerized instructions posted on the Internet[2], subjects were provided with some examples showing how to use the software and how to submit bids. Before each auction, subjects were informed of the support from which their values would be drawn and that, within the next ten auctions, they would be paired with a different participant. In every auction during the session, subjects were allowed to submit at most two bids for their two units and, after everybody had submitted their bids, the results were calculated and shown. Subjects could see only their bids and the bids of the person with whom they were matched. They were informed how many units they won, the prices they paid, and how much profit they had obtained from that auction. In the English auction, subjects were shown a price, which would increase every 10 seconds by a specified increment if total demand was greater than 2. They had to indicate the amount (either 0, 1 or 2) of units that they wanted at the posted price. Recall that subjects could not increase their quantity demanded from increment to increment. Table 2 reports the relevant information for each session.

Table 2

Experimental Design Summary

Date	**# of subjects**	**Auction**	**# of Auctions Conducted**
March 30, 1999	10	Uniform-Price	150
April 6,1999	10	Vickrey	150
April 7, 1999	10	Uniform-Price	150
April 8, 1999	10	Vickrey	150
September 19, 2000	10	English	150
September 21, 2000	10	English	150

IV. Results

<u>Result 1</u>:*Bids are not demand revealing in the Vickrey Auction*

<u>*Evidence*</u>*:* Figures 1 and 2 show the scatter diagram of bids for the high and low valued units. It is evident that very few of the bids can be classified as demand revealing. Each figure also shows the results of the regression:

[2] Instructions are available at http://economic.gmu.edu/experiments/roumen/index.html

bid$_{ij}$ = β (value$_{ij}$) + γ_1 (value support dummy$_1$)+ γ_2 (value support dummy$_2$)+ γ_3 (value support dummy$_3$) + ε_{ij}

where i denotes the bidder and j denotes that bidder's highest = 1 and lowest = 2 valued units. The value support dummy variables are for the ranges ([2, 50], [30, 170], [70, 260]) and are used to capture List and Reiley's conclusion that the value support affects the bids.

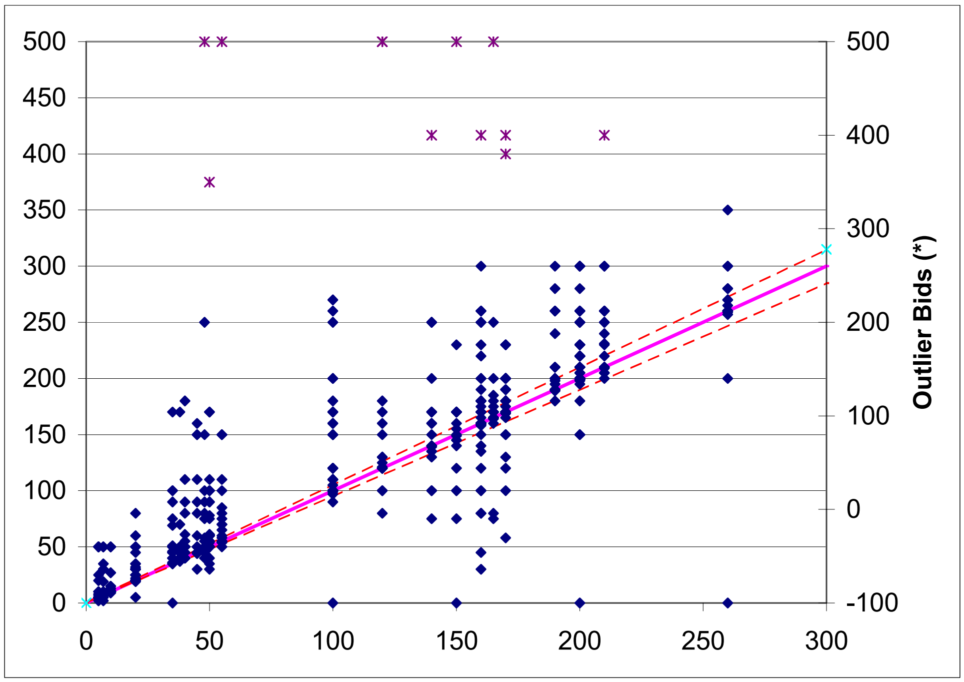

Figure 1.

Scatter Diagram of the bids on the high valued unit in the Vickrey Treatment, with values on the horizontal axis and bids on the vertical axis (the secondary y-axis shows outlier bids *) . The dense line is the 45 degree line. Dashed lines represent bids within 5% of their respective values. The regression estimate of the bid function is given below with standard errors of the estimates in parenthesis:

$$b_1(v_1) = \mathbf{0.85}v_1 + 18.83D_1 + 36.75D_2 + 52.54D_3 \qquad R^2 = 0.64$$

$$\quad (0.06) \qquad (4.36) \qquad (8.21) \qquad (11.27)$$

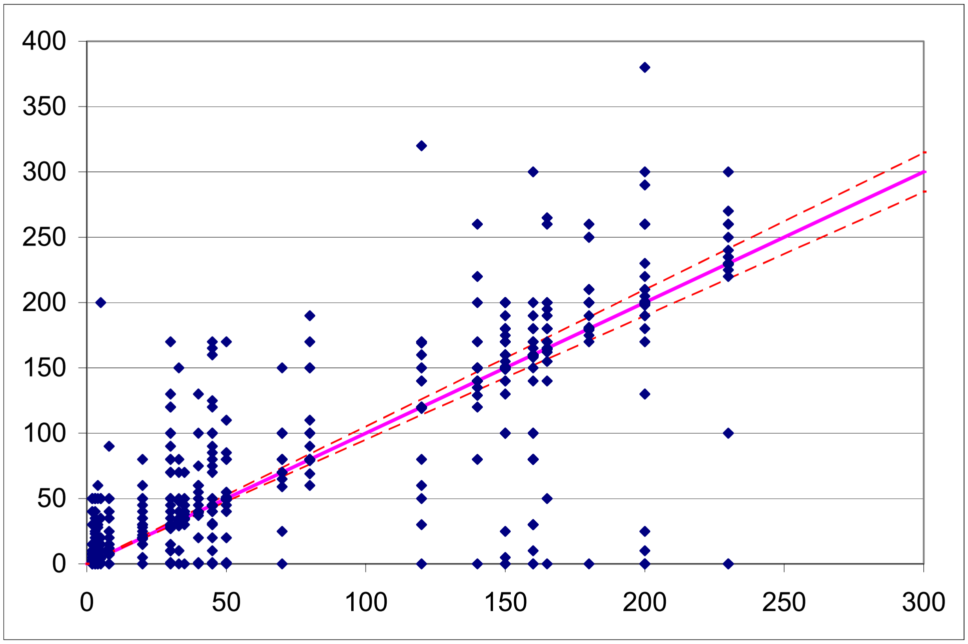

Figure 2.

Scatter diagram of the bids on the low valued unit in the Vickrey Treatment, with values on the horizontal axis and bids on the vertical axis. The dense line is the 45 degree line. Dashed lines represent bids within 5% of their respective values. The regression estimate of the bid function is given below with standard errors of the estimates in parenthesis:

$$b_2(v_2) = 0.74v_2 + 12.76D_1 + 22.43D_2 + 39.38D_3 \qquad R^2 = 0.64$$

$$(0.05) \quad (3.45) \quad (4.88) \quad (8.35)$$

We find that β is significantly less than 1 and the dummy variables are all significant (F statistic = 5.13) and large.[3] The dummy variable coefficients plus β point to over-bidding by participants on high valued units. Contrary to theory, the value support seems to be an important determinant of the over-bidding. Specifically, the higher the value, the greater is the tendency towards overbidding. For the low valued unit, β and the dummy variable coefficients are significant but smaller than the high valued estimates. Figure 3 shows the distribution of over- and under-revelation. We see that there is a consistent over-revelation of value in the bids, with only a quarter of the bids demand-revealing.

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[3] We tested several models including individual fixed effects and value support effects on the model coefficients. None of these more sophisticated models change the results stated here. The data from the experiments and the different statistical models we tested can be found at http://economic.gmu.edu/experiments/roumen/index.html

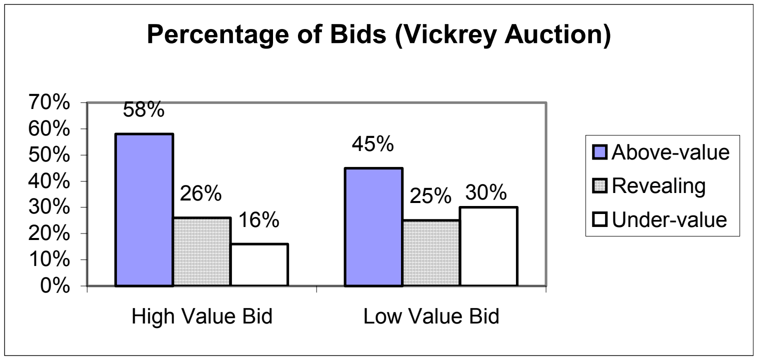

Figure 3.

Classification of bids in the Vickrey Treatment. Bids within 5% of value are categorized as revealing.

Result 1 differs from the results found by Cox et al. (1991) for single-unit second-price auctions. They find that subjects tend to bid over value initially but that this tendency disappears as the subjects gain experience. We hypothesize that learning occurs because overbidding can quickly result in a loss, which reinforces value bidding. With multi-unit demand, overbidding on the high valued units rarely results in a loss because the price is determined by the bids on the low valued units, which delays or eliminates the reinforcement mechanism.[4]

More importantly, Result 1 brings into question the reliance on Vickrey auction bids as a proxy for actual values when field participants have multi-unit demands. Considering our results, we would expect the sample average of the high bids in the field experiment to be an upward biased estimate of the bidders' values. The downward bias in the bids on the low valued unit disappears with the increase of the value support.

Result 2: Bids on the high valued unit are not demand revealing in the Uniform-price auction.

Evidence: Figure 4 shows the scatter diagram of bids for the high valued units in the Uniform-price auction. It is evident that few of the bids could be qualified as revealing. The figure also shows the results of the regression:

[4] We tested for differences in the estimate of the bid function for the first 15 auctions vs. the last 15 auctions. We found no statistically significant differences and thus, could not find evidence of bidders learning to bid their value.

$\text{bid}_{ij} = \beta\,(\text{value}_{ij}) + \gamma_1\,(\text{value support dummy}_1) + \gamma_2\,(\text{value support dummy}_2) + \gamma_3\,(\text{value support dummy}_3) + \varepsilon_{ij}$

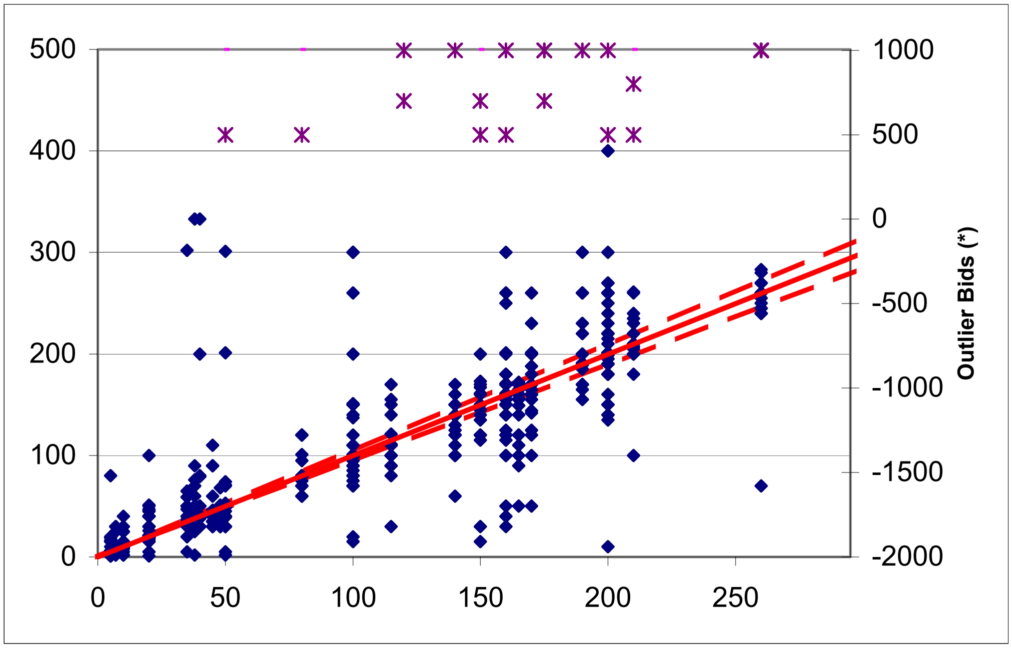

Figure 4.

Scatter diagram of the bids on the high valued unit in the Uniform-price treatment, with values on the horizontal axis and bids on the vertical axis. The dense line is the 45 degree line. Dashed lines represent bids within 5% of their respective values. The outlier bids (bids >500) are listed on the secondary y axis. The regression estimate of the bid function is given below with standard errors of the estimates in parenthesis.

$$b_1(v_1) = \underset{(0.15)}{1.12v_1} + \underset{(10.69)}{7.94D_1} + \underset{(20.63)}{7.42D_2} + \underset{(28.13)}{31.65D_3} \qquad R^2 = 0.31$$

The coefficient of the value is not significantly different from one but we notice that the number and the magnitude of the outlier bids is greater than in the Vickrey auction. This observation is consistent with previous experimental findings (see Kagel and Levin 1999). In about 10% of the auctions (28 out of 300 auctions) bidding over value resulted in a loss to a participant.

<u>Result 3</u>: Bids for low valued units in the Uniform-price auction are under revealing but they are not equal to the equilibrium prediction of 0.

Evidence: Figure 5 shows that in the Uniform-price auction approximately 68% of the bids for the low valued unit are below value.

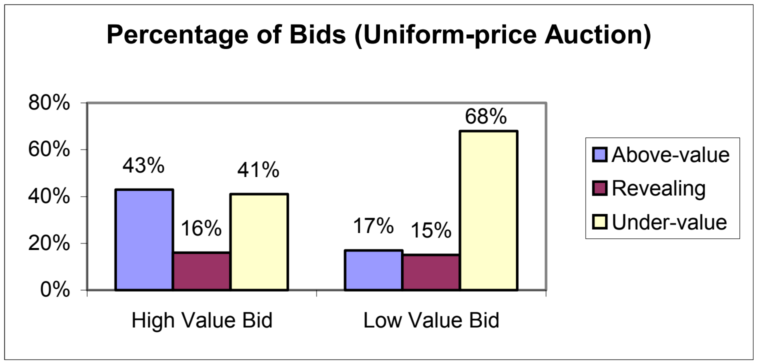

Figure 5.

Classification of bids in the Uniform-price Treatment. Bids within 5% of value are categorized as revealing.

Figure 6 shows the scatter diagram of bids in the Uniform-price auction for the low valued units. Under revealing is definitely more pronounced than on the high valued units or the low valued unit in the Vickrey auction, which is consistent with theory. The regression estimates on the bid function shows that $0 < \beta < 1$ and that the coefficients of the dummy variables are not significantly different from 0 (this result is also consistent with previous research -- see Kagel and Levin 1999, Alsemgeest et. al 1998.). Thus, in the Uniform-price auction we find that unlike the Vickrey auction the value supports do not matter and that demand reduction is prevalent.

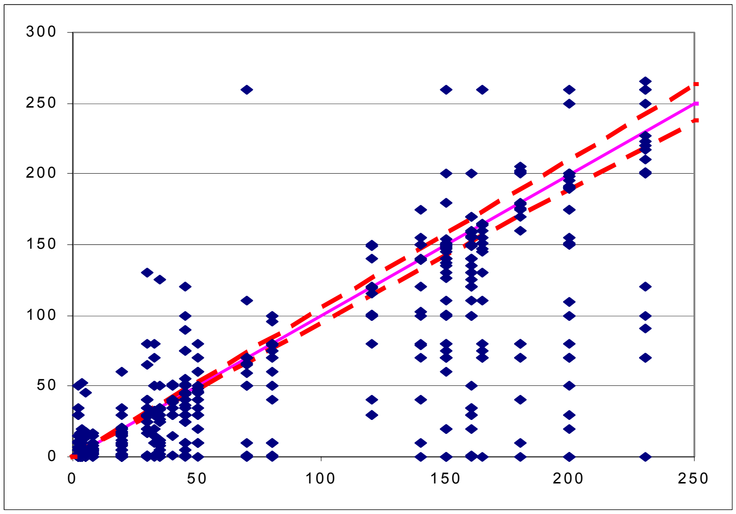

Figure 6.

Scatter diagram of the bids on the lower valued unit in the Uniform-price Treatment, with values on the horizontal axis and bids on the vertical axis. The dense line is the 45 degree line. Dashed lines represent bids within 5% of their respective values. The regression of the bid function is given below with standard errors of the estimates in parenthesis.

$$\mathbf{b_2(v_2) = 0.78v_2 + 0.86D_1 + 2.10D_2 + 2.64D_3} \qquad \mathbf{R^2 = 0.44}$$
$$(0.07) \quad (4.83) \quad (6.80) \quad (11.66)$$

Recall that List and Reiley (1997) find more "over-bidding" on high valued units in the Uniform-price auction than in the Vickrey bids. We consider the relative properties of the Uniform-price and Vickrey auctions next.

<u>Result 4</u>: Bids on the low-valued units are lower in the Uniform-price auction than the Vickrey auction.

<u>Evidence</u>: The proportion of bids under-value on the second unit in the Uniform-price treatment is 58%, while it is 24% in the Vickrey treatment. The magnitude of demand reduction is also greater in the Uniform-price treatment. To show this formally, in Figure 7 we plot the cumulative density of the percentage difference between value and bid, ***p*** ($\frac{value - bid}{value}$), and perform a standard two-sample t-test on the range (0.05,1.00]. The null hypothesis that there is no difference in the size of demand reduction can be rejected at the 5% confidence level.

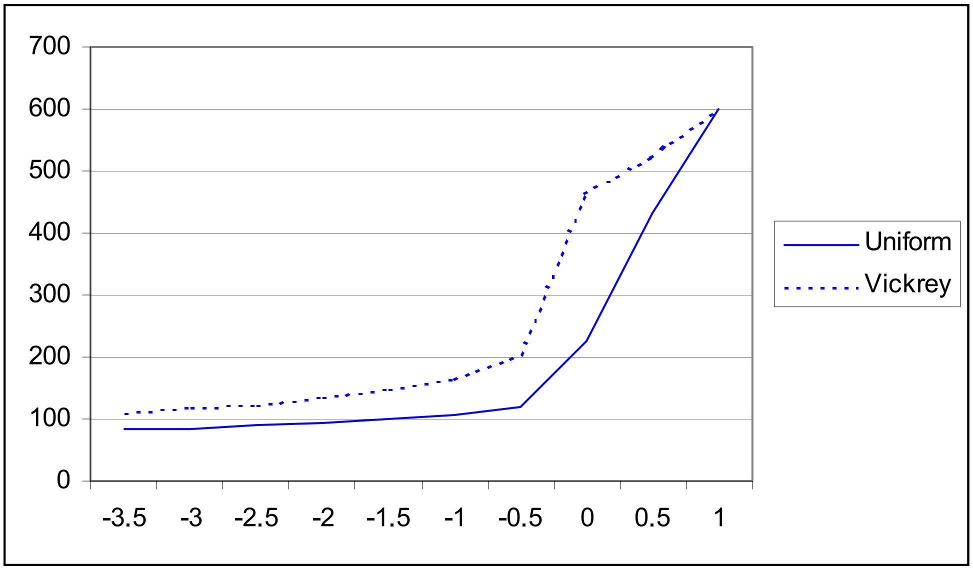

Figure 7.

Cumulative density function of the statistic $p = \frac{value - bid}{value}$ **for the bids on the low valued unit in the Uniform-price and Vickrey treatments. The number of bids is shown on the vertical axis**.

Result 5: Bids on the high valued units are higher in the Uniform-price auction than in the Vickrey auction.

Evidence: In Figure 8 we plot the cumulative density of the percentage difference between value and bid $\boldsymbol{p}$ ($\frac{value - bid}{value}$) and perform a standard two-sample t-test of $\boldsymbol{p}$ over the range $(-\infty, -0.05)$. The null hypothesis that there is no difference in the size of overbidding can be rejected at the 5% confidence level. This result is consistent with List and Reiley's second finding.

Result 6: Value support is important in the Vickrey auction, but not important in the Uniform-price auction.

Evidence: Both Uniform-price bid functions fail the joint F test of significance of dummy coefficients for the supports. Support dummy coefficients in the Vickrey bid functions are significant and increase with the increase in the support. Considering our regression results, we can say that the ratio $\frac{b_2^U(v)}{b_2^V(v)}$ seems to be a decreasing function of the relevant support. However, this decrease is more likely due to an increase in the denominator rather than to a decrease in the numerator. That is subjects tend to reveal or overbid more on higher values in the Vickrey auction rather than demand reduce more in the Uniform-price auction.

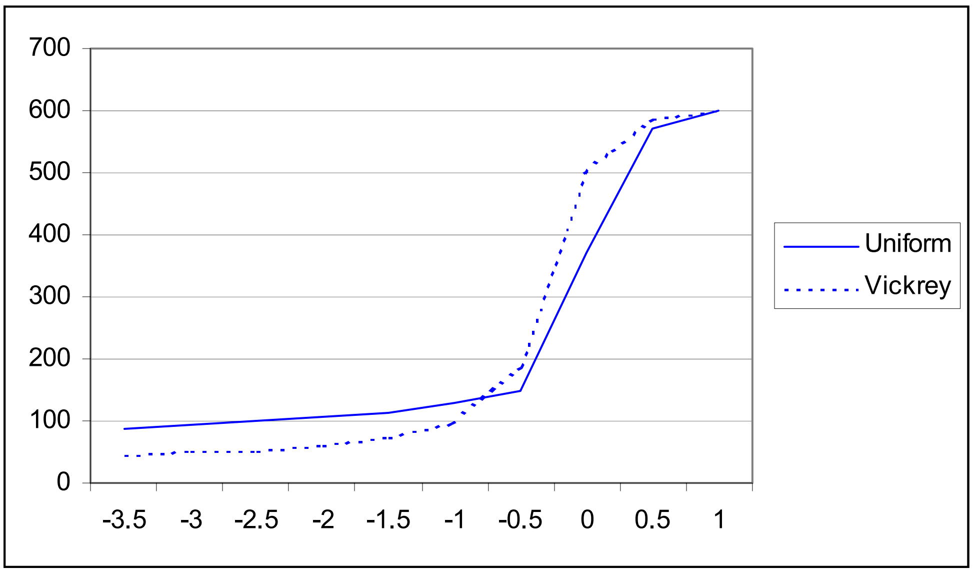

Figure 8.

Cumulative density function of the statistic $p = \frac{value - bid}{value}$ **for the bids on the high valued unit in the Uniform-price and Vickrey treatments. The number of bids is shown on the vertical axis.**

Recall that the English and Uniform-price auctions should be identical in terms of price and allocation. In the English auction, the prediction is that price should equal the initial price. While we can reject this strong prediction, we do find that the final price is very close to the opening price and is significantly below the price in the Uniform-price auctions.

<u>Result 7</u>: Price in the English auction is significantly below the price in the Uniform-price auction.

Evidence: Table 3 shows the average price (and standard deviation) for value draws from Appendix A. It is easy to see that prices in the Uniform-price auction are consistently higher than the prices in the English auction and that prices in the Uniform-price auction are not significantly different from the prices in the Vickrey auction.

Table 3

Average Price per unit (Standard Deviation) for selected Value Draws across Uniform-price and English Auctions.

Value Draw	Uniform-price	English	Vickrey
(160, 160), (48, 33)	62.61 (41.18)	14.00 (13.00)	60.43 (47.32)
(150, 45), (160,160)	84.50 (68.43)	22.00 (18.02)	72.50 (35.93)
(200, 200), (100, 80)	88.50 (25.14)	27.50 (17.12)	117.80 (44.00)
(20, 20), (50, 5)	8.82 (7.30)	7.00 (5.55)	17.00 (12.12)
(160, 160), (140, 30)	112.5 (40.21)	20.00 (14.31)	106.08 (38.85)
(200, 200), (160, 140)	115.00 (36.60)	45.00 (43.24)	152.83 (36.62)
(50, 4), (20, 20)	15.16 (10.62)	6.00 (2.19)	15.33 (5.05)
(100, 70), (200, 200)	105.12 (43.50)	42.50 (3.53)	84.31 (58.01)

An obvious corollary to result 7 is the effect on revenues, which we supply next.

Result 8: Revenue can be ranked as follows: $R^V(v_1, v_2, v_3, v_4) = R^U(v_1, v_2, v_3, v_4) > R^E(v_1, v_2, v_3, v_4)$

Evidence: We cannot reject the hypothesis that there is a difference in the revenue between the Vickrey and the Uniform-price auction, although in total there seems to be some downward revenue bias in the Uniform-price auction. Revenue in the English auction is significantly lower than the revenue in the Uniform-price auction for 76% of all draws.

A potential reason for the stark difference between the English auction and a sealed bid auction in this environment is the fact that in the Uniform-price auction subjects do not act consistently with equilibrium play. In the English auctions subjects realize much more quickly to reduce demand. The transparency of when to reduce demand offered in the English auction allows subjects to learn that the longer they stay in the auction, the less their expected profit will be.[5] This is in contrast to the uncertainty of the behavior of others in the Uniform-price auction that tempers demand reduction. Turning to efficiency we find:

Result 9: Efficiency can be ranked as follows: $E^V(v_1, v_2, v_3, v_4) = E^U(v_1, v_2, v_3, v_4) > E^E(v_1, v_2, v_3, v_4)$

Evidence: Table 4 shows that the efficiency outcomes of the auctions are closely related to the participants' values and are far from the theoretical predictions. When the two bidders share the two highest values, the Uniform-price and English auctions are more efficient than the Vickrey auction at the 5% confidence level. This is somewhat reversed in the case when one of the bidders has the two highest values. Over all possible combinations, the efficiency of the two sealed-bid auctions is equal and surpasses that of the English auction.

Table 4

Percentage of allocations that are efficient across Vickrey, Uniform-price and English Auctions

Value Draw	Vickrey	Uniform-price	English
(160, 160), (48, 33)	88%	50%	25%
(150, 45), (160,160)	0%	0%	0%
(200, 200), (100, 80)	75%	50%	0%
(20, 20), (50, 5)	75%	100%	100%
(160, 160), (140, 30)	33%	33%	17%
(200, 200), (160, 140)	67%	50%	0%
(50, 4), (20, 20)	83%	100%	100%
(100, 70), (200, 200)	75%	63%	0%
Total (All auctions)	70%	68%	50%

[5] We decided to specifically test for learning effects in our data. There is no significant difference between the bids in the first fifteen periods of the experiment compared to the last fifteen periods for the Vickrey and the Uniform-price auctions. In the English auction, however, subjects learn to use their dominant strategy during the course of the experiment. In the following regression equation

$$\mathbf{P_n} = \underset{(0.34)}{3.33\mathbf{I_n}} + \underset{(2.43)}{5.79\mathbf{L_1}} - \underset{(2.42)}{7.54\mathbf{L_2}} \qquad \mathbf{R^2 = 0.33}$$

$\mathbf{P_n}$ is the price per unit in the nth auction, $\mathbf{I_n}$ is the initial price, $\mathbf{L_1}$ and $\mathbf{L_2}$ are dummy variables pertaining to the first and second half of the experiment. We find that price (and revenue) is significantly less (closer to the theoretical prediction) in the second half of the experiment (F statistic = 49.14)

V. Conclusion

Our laboratory replication of the field experiments of List and Reiley has uncovered the apparent inconsistency they found that "demand reduction is evident in the Uniform-price auctions, relative to the Vickrey auctions… but first-unit bids are higher in Uniform-price than in Vickrey auctions." The phenomenon occurs because there is a substantial deviation from demand revelation in both the Vickrey and Uniform-price auctions. A reliance on the Vickrey auction to obtain estimates of true value may not be reasonable when participants have multi-unit demands. However, it is clear that the Uniform-price auction provides greater incentives to reduce demand than does the Vickrey auction. Moreover, the English auction provides the greatest incentive to demand reduce, which leads to lower revenues and efficiency than in both the Uniform-price and Vickrey auctions. We hypothesize that the low information content in sealed bid auctions tempers the amount of demand reduction that actually occurs. Thus, the concerns raised by Charles River and Associates (CRA 1999) for potential demand reduction in the FCC spectrum auctions is warranted.

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Appendix A: Value Draws

Auction 1-10	Auction 11-20	Auction 21-30
150,45	200,150	10,2
120,50	190,165	7,3
160,160	200,200	20,20
160,160	200,200	20,20
170,120	260,230	38,35
45,30	210,180	5,2
165,45	170,150	40,8
140,30	160,140	35,3
55,40	100,80	50,5
48,33	100,70	50,4

Appendix B: Symmetric Bayesian Equilibrium in an English Auction for Two Units

Values are drawn uniformly from [a,b]. Without loss of generality, suppose that at some price c < v_2 in the auction, bidder two drops his demand from 2 to 1. Bidder one, with values v_1 and v_2 now has two options. The first option is to drop his own demand to 1 in order to end the auction and get a profit $\Pi_1 = v_1 - c$. The second option is to continue demanding two units until he reaches v_2 or until his opponent drops demand to 0. Expected profit for bidder one in this case is

$$\Pi_2 = (v_1 - v_2).\int_{v_2}^{b} h(x \mid v_2^o = c)dx + \int_c^{v_2} (v_1 + v_2 - 2x)h(x \mid v_2^o = c)dx, \qquad (1)$$

where

$$h(x \mid v_2^o = c) = \frac{2}{(b-a)^2}(x-a) \qquad (2)$$

h(.) is the density. of bidder two's high value, conditional on his low value being c.
If we hold v_1 and c fixed, then we find that

$$\frac{\partial}{\partial v_2}\Pi_2(v_2) = \int_c^{v_2} h(x \mid v_2^o = c)dx - \int_{v_2}^{b} h(x \mid v_2^o = c)dx. \qquad (3)$$

Now it is clear that in the interval [c,v_1] the function Π_2 initially decreases, then reaches its minimum where $\frac{\partial}{\partial v_2}\Pi_2(v_2) = 0$ then increases again. (Note that h(.) is a linear, strictly increasing function). Therefore, the maximum, must be at one of the end points. We have

$$\Pi_2(c) = v_1 - c \qquad (4)$$

and

$$\Pi_2(v_1) = 2v_1\int_c^{v_1} h(x \mid v_2^o = c)dx - 2\int_c^{v_1} xh(x \mid v_2^o = c)dx \qquad (5)$$

Just using the properties of the density h(.) we can say that the highest possible value of the first expression in (5) is $2v_1$. It is also obvious that

$$2\int_c^{v_1} xh(x \mid v_2^o = c)dx > v_1 + c$$

Thus, $\Pi_2(c) > \Pi_2(v_1)$, so (1) achieves its maximum in the interval [c, v_1] only when $v_2 = c$. So $\Pi_1 \geq \Pi_2$. for all v_1, v_2, and c.
Therefore, bidder one has an incentive to decrease his demand to one immediately after bidder two has done the same.
Generally, any strategy, for which both bidders drop one unit at some price less than the lowest value and one unit at their highest value, is a part of a symmetric equilibrium. Clearly dropping at 0 maximizes the profit of both bidders in expected terms. It also maximizes gains irrespective of what the other bidder does. Therefore, dropping at 0 is a weakly dominant strategy.

Appendix C: Symmetric Bayes-Nash Equilibrium for Uniform-price Sealed Bid Auction for 2 Units

Suppose there are two bidders in the auction. Each of them has values for two units chosen independently and uniformly from [a,b].To find the equilibrium bid function first we suppose that there exist F_1^* and F_2^* ,which are, respectively, the cumulative distributions of the lower and higher bids in equilibrium.

The expected profit for a bidder with values v_1 and v_2 and bids b_1 and b_2 is

$$\int_{b_2}^{b_1}(v_1-x)f_1^*(x)dx+\int_a^{b_2}(v_1+v_2-2x)f_2^*(x)dx+(v_1-b_2)(F_1^*(b_2)-F_2^*(b_2))$$

The First Order Necessary Conditions are:

$$(v_1-b_1)f_1^*(b_1)=0\,;\ b_1>0 \text{ or} \qquad (6)$$
$$(v_1-b_1)f_1^*(b_1)\le 0\,;\ b_1=0$$

and

$$(v_2-b_2)f_2^*(b_2)-(F_1^*(b_2)-F_2^*(b_2))=0\,;\ b_2>0 \text{ or} \qquad (7)$$
$$(v_2-b_2)f_2^*(b_2)-(F_1^*(b_2)-F_2^*(b_2))\le 0\,;\ b_2=0$$

From (6) we have $b_1=v_1$ because $f_1^*(b_1)>0$, i.e. the dominant strategy is to reveal on the first bid. This suggests that the first bid in equilibrium has the same distribution as the highest of the bidder's values.

Therefore

$F_2^*(b_2)=(\frac{b_2-a}{b-a})^2$ and $f_2^*(b_2)=\frac{2}{(b-a)^2}(b_2-a)$.

Substituting into (7) we get

$$F_1^*(b_2)=\frac{(b_2-a)}{(b-a)^2}(2v_2-b_2-a) \qquad (8)$$

Now we suppose that for every $b_2>0$ there is a well defined strictly increasing inverse bid function $v_2(b_2)$. We have

$\Pr[x\le b_2]=\Pr[v_2(x)\le v_2(b_2)]$

or

$$F_1^*(b_2)=1-\left(\frac{b-v_2(b_2)}{b-a}\right)^2 \qquad (9)$$

Substituting (8) into (9) and solving for b_2 yields

$b_2=v_2\pm\sqrt{2v_2^2-2v_2(a+b)+2ab}$.

We notice that b_2 is not a real number if $v_2\in[a,b]$. The only real solution to the FOC is:

$b_1=v_1$

and

$b_2=0$

which is the Symmetric Bayes-Nash Equilibrium of the auction.